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Letter to the Editor

Reply to Comments by F.M. Detinko on the "Finite Element Solution of the Stability Problem for Nonlinear Undamped and Damped Systems Under Nonconservative Loading", *Int. J. Solids Structures* Vol. 34, No. 19, pp. 2497–2516 (1997) by R.V. Vitaliani, A.M. Gasparini and A.V. Saetta

We agree with F. M. Detinko that the verification of the finite element results of nonlinear problems, like those presented in our paper 'Finite element solution of the stability problem for nonlinear undamped and damped systems under nonconservative loading', is a very interesting matter, especially when the problems are intended to serve as benchmark tests. Therefore the contribution of Dr Detinko's discussion is of great concern, even if it is limited to an analytical check of the local-displacement diagrams, and no information on the stability behaviour or critical load, can be inferred.

Regarding this question, we would like to underline that the aim of our paper was the study of the stability conditions in terms of type of critical load and effect of damping for the proposed benchmarks. As a consequence, the main results obtained by using the numerical approach developed in the paper are:

- the value and the type of critical load (divergence or flutter);
- the effect of damping.

However, in order to give a more precise comparison between our results and those obtained by Dr Detinko, we have replotted the load-displacement diagrams of problems B5 and B13 and marked the points of the analytical solutions (drawn from Tables 2 and 3 of Dr Detinko's discussion) (see Figs. 1 and 2). It is worth noting that the two approaches correspond well, even for near critical loads.

With respect to the third example, B18, we would like to thank Dr Detinko for pointing out the typographical error that was present in our paper. The published value for the Young's modulus was $E = 7.124 \times 10^7$ N/cm^2, instead of the correct value $E = 7.124 \times 10^6$ N/cm^2. The published results regarding this example, however, are those obtained by using the correct value of modulus E. Therefore the comparison between the displacements calculated with linear approximation and those obtained with the numerical approach has to be performed for a load lower than $P = 1$ kN, since the displacements under this load cannot be considered small.

The following table summarises the results obtained under a load $P = 0.1$ kN, calculated with the exact value of the Young's modulus. A good compatibility between the two approaches can be observed.

	Linear approximation	Numerical approach
u_x/L	0.080	0.07596
u_y/L	0.030	0.02472
φ	0.090	0.08835

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However, we would like to underline that the differences between our results and those of Argyris and Symeondis (1981a) specifically concern the deflected shape for the load value greater than $p = 3$ kN. In other words, such differences do not take into account the starting point of the load-displacement curve, where the two numerical solutions are almost coincident.

Regarding example B13, the difference underlined by Dr Detinko between our results and the analytical solution, concerns only the very small value of displacements near the critical value of applied load. Such a difference, even if not negligible in absolute value, has no great significance in terms of the global behaviour of the beam (e.g. load-displacement curve) (Fig. 2), where no appreciable difference can be observed.

Therefore, we can conclude that the load-displacement curves obtained with the developed finite element procedure do not appreciably differ from the closed form solution either for the small value of applied loads, or for near critical loads, unless the displacements are very small. In this case we feel that the difference in absolute value can be considered irrelevant.

Finally, we would like to thank Dr Detinko for his very interesting discussion and especially for having supplied the possibility of an analytical check of our results.

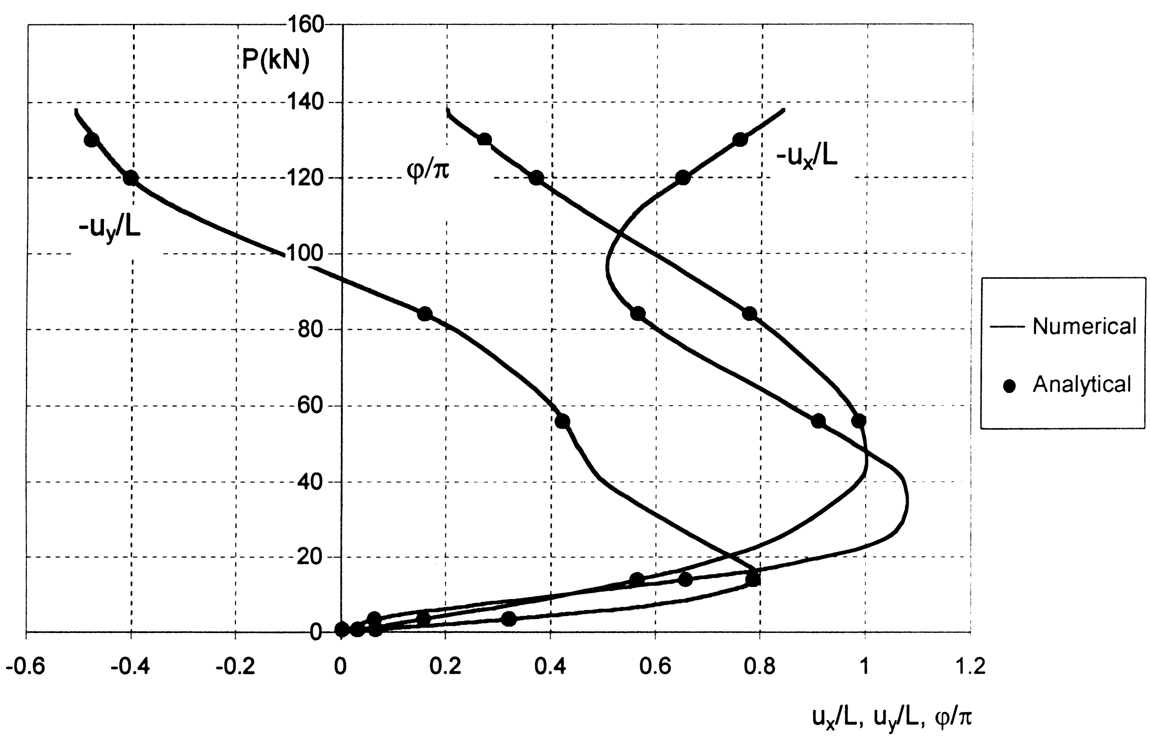

Fig. 1. Comparison between analytical and numerical results for a cantilever under nonconservative end loads (B5).

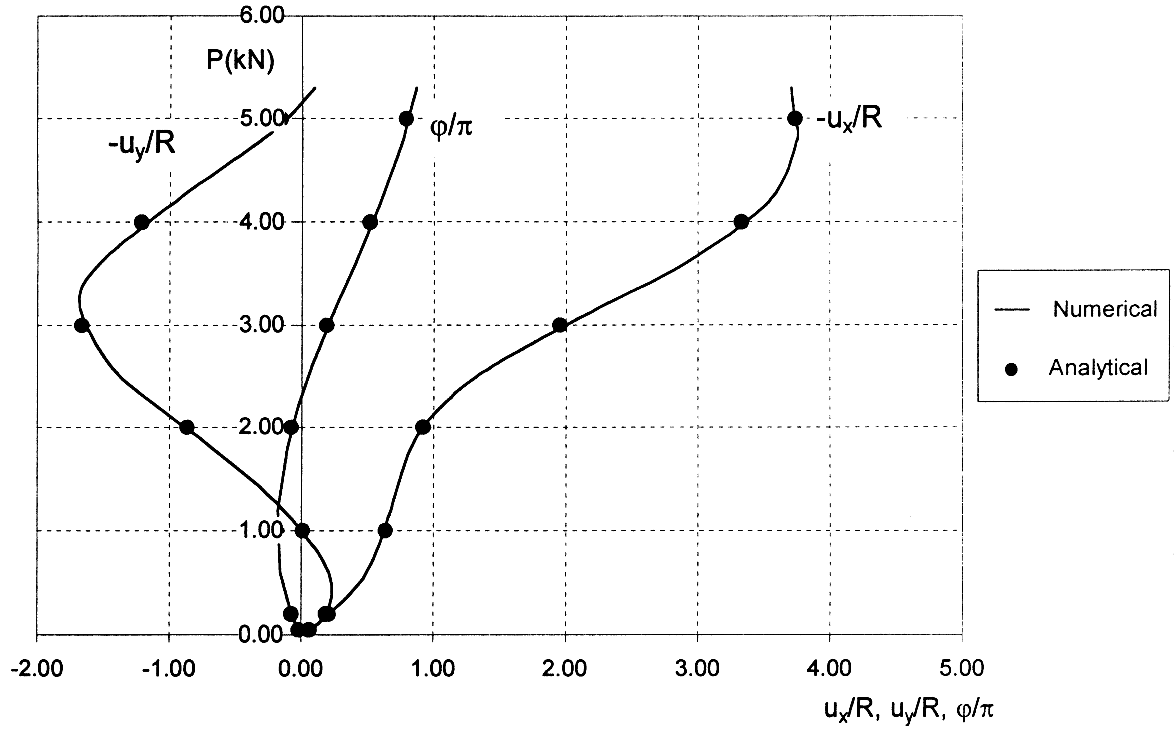

Fig. 2. Comparison between analytical and numerical results for a curved cantilever under nonconservative end loads (B13).

Reference

Argyris, J.H., Symeondis, S., 1981a. A sequel to: nonlinear finite element analysis of elastic systems under nonconservative loading. Natural formulation. Part I. Quasistatic problem. Comp. Math. Appl. Mech. Engng. 26, 377–383.

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